



Thermal Flux and Magnetic Field Effects on Nano-fluids Over Shrinking/Stretching Sheet

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ABSTRACT

The motivation for studying nanofluid behavior under the influence of various external forces stems from its numerous applications in a variety of engineering industries. This paper focuses on the effect of a magnetic field and heat flux on a Shrinking / Stretching Sheet in a base fluid. Nonlinear partial differential equations (PDEs) are reduced to nonlinear ordinary differential equations (ODEs) via a similarity transformation, which are then numerically solved for two types of nanoparticles, namely copper and alumina, in the water-based fluid using the shooting technique. Considering the steady two-dimensional stagnation-point flow of a water-based nanofluid over a stretching/shrinking sheet. For some of the governing parameters of Grashof number (Gr), magnetic field (M), Prandtl number (Pr), and volume fraction (ϕ), the velocity and temperature profiles were presented graphically and thoroughly discussed. It found that magnetic parameters and Grashof number solutions for a shrinking/stretching sheet increase velocity while decreasing temperature.

Keywords: nanofluids, stagnation-point flow, Grashof number, magnetic field, stretching /shrinking sheet.

1. Introduction

Fully understanding the behavior of nanofluids influenced by various social forces is significant since it is applied in several fields of engineering. Depending on the nanoparticles, nanofluids possess variable physical and chemical properties. The terms of Nanofluid by Choi *et al.* (2001). They demonstrated that adding small amounts of nanoparticles to the base fluid raises the fluid's thermal conductivity by at least twice its thermal prosperity. This attitude prompted researchers to investigate new generation fluids by incorporating nano-sized solid particles into conventional fluids to improve and tune the thermophysical properties of Khairul *et al.* (2017). In both cases, a fixed or moving body is in a fluid. In a nanofluid describing fluid motion in the stagnation area of a solid surface, the effect of a magnetic field and heat flow over a stretching/shrinking sheet exists. Stagnation-point flow over a stretching/shrinking sheet in a nanofluid has been considered by Bachok *et al.* (2011), it is estimated that conventional fluids are incapable of supporting more sophisticated engineering designs. Magnetic nanofluids are colloids made up of nano-sized ferromagnetic particles dispersed in a base fluid; surfactants may be added to keep the mixtures stable. Due to their importance in several technical applications such as purification of molten metals, cooling of nuclear reactors, and others, magnetic field effects on fluid flow and heat transfer have gotten a lot of attention. Oztop *et al.* (2011) studied mixed convection with a magnetic field in a top-sided lid-driven cavity heated by a corner heater. Hiemenz, (1911) created the first study in this field. He used similarity transformation to reduce two-dimensional Navier–Stokes equations to a nonlinear ordinary differential equation (ODEs) and then presented the exact solution. A similar solution by Homann and Angew, (1936) was used to extend

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this research to the case of axisymmetric three-dimensional stagnation point flow. Following these pioneering studies, many researchers have focused their efforts on this topic. For this topic, Attia, (2003) studied the effects of an external magnetic field in the presence of suction/blowing parameters. Later, Massoudi and Ramezan, (1990) used a perturbation technique to solve the stagnation point flow and heat transfer equation of a non-Newtonian second-grade fluid. They later reviewed their previous research in the case of non-isothermal surfaces, Massoudi and Ramezan, (1992). Garg, (1994) numerically improved Massoudi's results for any non-Newtonian parameter value. Malvandi, (2013) has some excellent literature in this area. The unsteady stagnation point flow of a nanofluid over a stretched sheet with slip effects was investigated by F. Hedayati *et al.* (2014). Free magnetohydrodynamic convection has a variety of uses, including cooling electronic components, combustion modeling, and fire engineering. Recently, nanotechnology has provided a novel passive method for improving heat transmission. Alsabery *et al.* (2016) studied nanofluid conjugate free convection with sinusoidal temperature variation. Mahapatra and Gupta, (2002, 2003); considered the combination of both stagnation-point flows past a stretching surface. There are two conditions under which flow towards a shrinking sheet is likely to exist, depending on whether an adequate suction on the boundary is imposed by Miklavčič and Wang (2006). Wang, (2008) investigated both two-dimensional and axisymmetric stagnation flow towards a shrinking sheet in a viscous fluid. Following this pioneering work, the flow field over a stagnation point towards a stretching/shrinking sheet has gotten a lot of attention, and a lot of literature has been generated on the subject, Lok *et al.* (2011). All of the preceding research applies to the stagnation point flow towards a stretching/shrinking sheet in a viscous and Newtonian fluid. The issue of a steady boundary-layer flow, heat transfer, magnetic parameters, and nanoparticle fraction across a stagnation point towards a stretching/shrinking sheet in a nanofluid, using water as the basis fluid, is addressed in this research. The low thermal characteristics of the majority of common heat transfer fluids, like water, ethylene glycol, and engine oil, can restrict their usage in many thermal applications. The majority of solids, metals, and magnetohydrodynamics, in particular, have a wide range of real-world uses in engineering and technology, such as crystal growth, liquid-metal cooling of reactors, plasma, magnetohydrodynamic sensors, electromagnetic casting, MHD power generation, and magnetic drug targeting. Magnetohydrodynamics is impacted by magnetic induction strength. The current paper considers a problem using Tiwari and Das's, (2002) nanofluid model and presents the effect of a magnetic field and heat flux over a stretching /shrinking sheet in nanofluid to convert the partial differential equations to an ordinary differential equation by using a similarity transformation, which is solved numerically by shooting method for two types of nanoparticles, namely copper (Cu) and alumina (Al_2O_3) in the water-based fluid. It was discovered that for a stretching/ shrinking sheet, magnetic parameters and Grashof number solutions increase velocity while decreasing temperature.

2. Materials and Methods (Model Proposal)

Generally, through this article, the nano-fluid will be assumed to be an incompressible nanofluid in the region $y > 0$ driven by a stretching/shrinking surface at $y = 0$ with a fixed stagnation point at $x = 0$, as shown in Figure 1. The stretching/shrinking velocity $U_w(x)$ and the ambient fluid velocity

$U_\infty(x)$ are assumed to vary linearly from the stagnation point, i.e. $U_w(x) = ax$ and $U_\infty(x) = bx$, where a and b are constants with $b > 0$. We notice that $a > 0$ and $a < 0$ correspond to stretching and shrinking sheets, respectively. The boundary layer flow of a steady, laminar, and incompressible nanofluid is governed by simplified two-dimensional equations

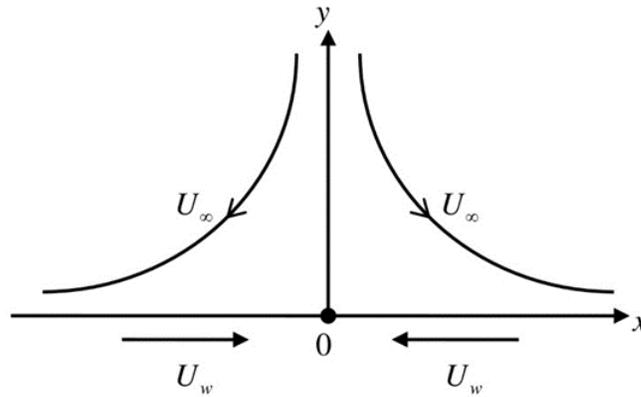


Figure 1: Physical model and coordinate system.

$$u_x + v_y = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} - \frac{\sigma_f B_o^2}{\rho_{nf}} u = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}} u_{yy} + (1-\phi)gB_{nf}(T-T_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} T_{yy} \tag{3}$$

Refer to the boundary conditions

$$\begin{aligned} \text{at } y = 0, \quad & u = U_w(x), \quad v = 0, \quad T = T_w \\ \text{as } y \rightarrow \infty, \quad & u = U_\infty(x), \quad T = T_\infty \end{aligned} \tag{4}$$

Where

* $u = u(x,y)$ is the component of velocity in the x direction

* $v = v(x,y)$ is the component of velocity in y direction

* T is the temperature of the nanofluid

The density of the nanofluid $\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s$

, the thermal diffusivity of the nanofluid $\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$,

and The viscosity of the nanofluid $\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}$, (5)

which

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}, \quad (\rho C_p)_{nf} = (1 - \varphi) (\rho C_p)_f + \varphi (\rho C_p)_s$$

Here,

φ is the nanoparticle volume fraction,

$(\rho C_p)_{nf}$ is the heat capacity of the nanofluid,

k_{nf} is the thermal conductivity of the nanofluid,

k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively,

ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively.

Similarity transformation

$$\psi = (v_f b)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \left(\frac{b}{v_f}\right)^{1/2} y \quad (6)$$

The satisfaction of the continuity equation is obvious, where ψ is the stream function defined as $u = \partial \psi / \partial y$, v is a negative partial derivative of a stream function with respect to x ,

$v = -\partial \psi / \partial x$, and η is the similarity variable. Using similarities (5)-(6) to Eqs. (1)-(3) and the boundary conditions (4), to transform into the dimensionless equations:

$$\frac{1}{(1 - \varphi)^{2.5} (1 - \varphi + \varphi \rho_s / \rho_f)} f''' + f f'' - f'^2 + M f' + (1 - \varphi) Gr \theta + 1 = 0 \quad (7)$$

$$\frac{1}{Pr \left[1 - \varphi + \varphi (\rho C_p)_s / (\rho C_p)_f \right] \frac{k_{nf} / k_f}}{\theta'' + f \theta'} = 0 \quad (8)$$

subjected to the boundary conditions (4) which become :

$$\begin{aligned} f(0) = 0, \quad f'(0) = \lambda, \quad \theta(0) = 1, \\ f'(\eta) = 1, \quad \theta(\eta) = 0 \quad \text{at } \eta \rightarrow \infty, \end{aligned} \quad (9)$$

After simplification, the dimensional physical parameters are defined as:

$$M = \frac{\sigma_f B^2}{b \rho_f (1 - \varphi + \varphi \rho_s / \rho_f)}, Gr = \frac{g B_{nf} (T_w - T_\infty)}{x b^2}, Pr = \left(\frac{\mu}{\nu \rho}\right)_f,$$

λ is the velocity ratio parameter defined as $\lambda = \frac{a}{b}$ where $\lambda > 0$ for stretching and $\lambda < 0$ for shrinking.

M is a magnetic parameter, Gr is the Grashof number, Pr is the Prandtl number.

Nomenclature

- x, y Dimensionless coordinates.
- u, v Velocity components along x-y axes.
- φ nanoparticle of volume fraction
- g gravitational acceleration
- a, b constants.
- M Magnetic parameter.
- C_p Specific heat
- Pr Prandtl number
- T fluid temperature
- C_p Specific heat
- Gr Grashof number
- T_∞ the fluid temperature of the ambient fluid
- U_∞ free stream velocity
- U_w ambient fluid velocity
- $(\rho C_p)_{nf}$ heat capacitance of the nanofluid
- B magnetic field

Greek Letters

- ρ fluid density
- μ dynamic viscosity
- α thermal diffusivity
- k thermal conductivity
- ψ stream function
- η similarity variable
- θ dimensionless temperature
- σ electrical conductivity
- λ constant mixed convection parameter

Subscripts

- f** fluid fraction
- s** solid fraction
- nf** nanofluid fraction

3. Results and Discussion

Using the shooting method, numerical solutions to the governing (ODEs) (7) and (8) with the boundary conditions (9) were obtained.

In this method, firstly it demonstrated the impact of the first parameter on which our research paper is based, we investigate the effects of the Magnetic parameter (M), Grashof number (Gr), the solid volume fraction of nanoparticles (φ), and Prandtl number Pr , for stretching/shrinking sheet.

parameters are analyzed for two different nanoparticles, namely (Cu–water) and (Al₂O₃– water). The value of the Prandtl number Pr is taken as 6.2 (water) and the volume fraction of nanoparticles is from 0 to 0.2 ($0 \leq \varphi \leq 0.2$) in which $\varphi = 0$ corresponds to the regular fluid. The thermophysical properties of the base fluid and the nanoparticles are listed in Table 1.

Table 1 Fluid and nanoparticle thermophysical properties:

Physical Properties	Fluid Phase (water)	Cu	Al ₂ O ₃
ρ (kg/ m ³)	9971.1	8933	3970
c_p (J/kg K)	4179	385	765
k (W/mK)	0.613	400	40

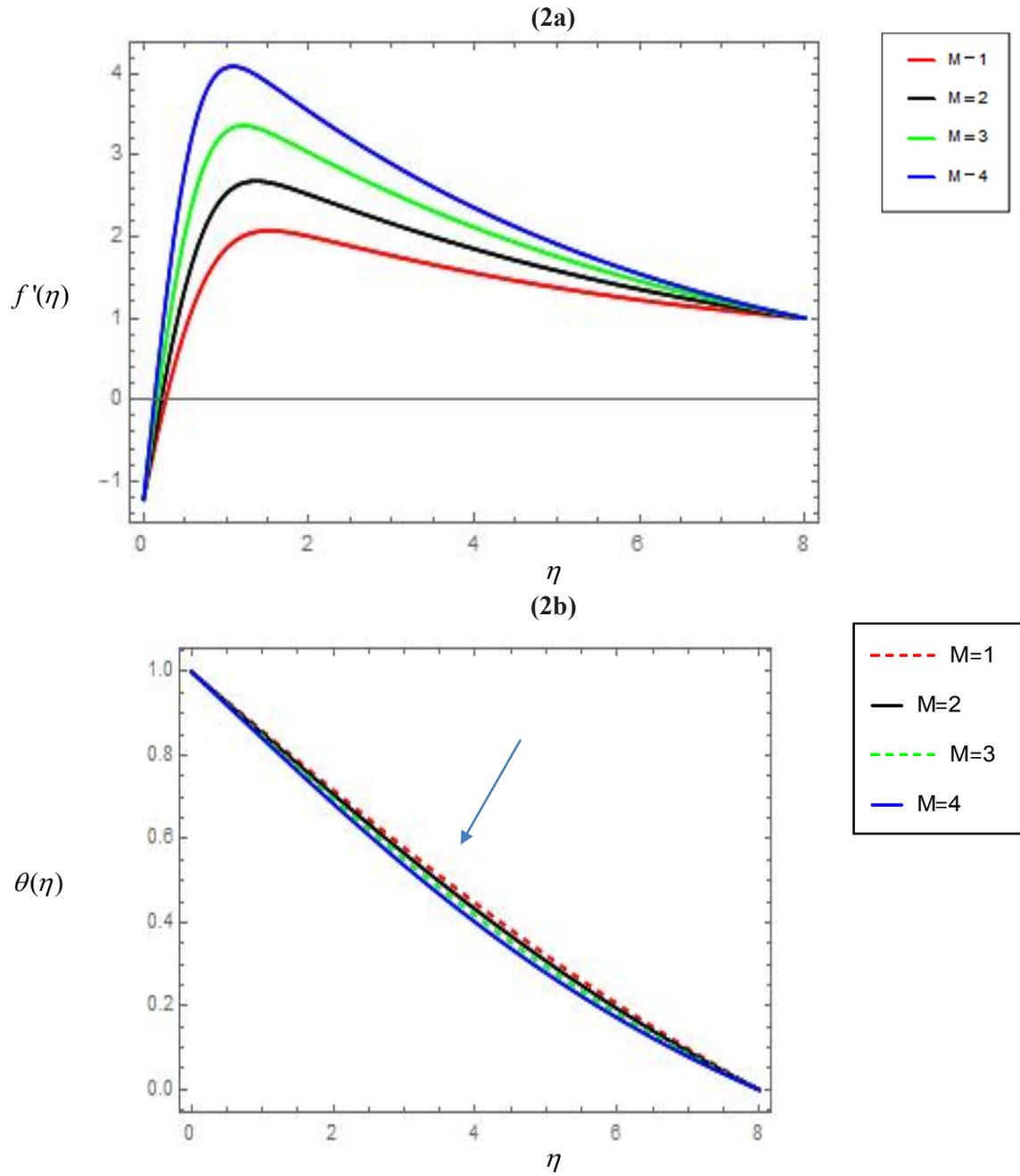
Figures show the transient and steady-state profiles of fluid velocity and temperature:

For shrinking sheet with parameters

M=1,2,3,4 with $\varphi=0.2$, Pr=6.2, Gr=3 and $\lambda=-1.22$. Fig. 2a shows how M affects the velocity and temperature profiles of (cu-water); as the magnetic parameter M is raised, the velocity profile rises. As seen in Fig. 2b, as the magnetic parameter M is increased, the temperature profile of (cu-water) drops. The effect of M on the velocity profile is shown in Fig 3a (Al₂O₃-water), when the magnetic parameter M is increased, the velocity profile increases. The temperature profile for (Al₂O₃-water) is shown in Fig 3b, as the magnetic parameter M is increased, the temperature profile reduces. In fig 4a for shrinking sheet, the effect of grashof number Gr=1,10,30. for (cu-water) with $\varphi=0.2$, $\lambda = -1.22$, pr = 6.2 and M=3. When the grashof number Gr is raised, the velocity profile rises. The influence on the temperature profile for (cu-water) is shown in Fig. 4b. As the Grashof number (Gr) is raised, the temperature profile drops. Figs 5a,5b for shrinking sheet, effect of grashof number Gr=5,10,15. for Al₂O₃ (-water), with $\varphi=0.2$, $\lambda = -1.22$, pr = 6.2 and M=3. In fig 5a, the velocity profile increases when the grashof number Gr is increased. Fig 5b shows the influence on the temperature profile for (Al₂O₃-water) **for stretching sheets with** $\lambda=2$, $\varphi=0.2$, pr = 6.2. Figures 6a, 6b,7a, 7b, 8a, 8b, 9a, 9b show the effects of (Al₂O₃-water) and (cu-water), respectively. As the magnetic parameters (M) and (Gr) are raised, it can be seen that the velocity rises and the temperature falls. The Grashof number demonstrates how buoyant forces affect natural convection. $Gr > 0$ indicates increasing field densities, which results in a higher buoyancy force. A higher Grashof number creates greater secondary flows due to convection and raises the flow velocity through the channel.

For shrinking sheet with φ (0, 0.1, 0.2)

The effects of different volume fractions of φ (0, 0.1, 0.2), solid nanoparticles on axial velocity field variations in a constant section were studied. The goal was to investigate the effect of changing the solid nanoparticle volume fraction on the fluid's axial velocity field. Because the plate is stationary and there is no slip boundary condition, the velocity of the nanofluid is zero at solid walls. Based on the behavior of curves in Fig 10a(cu-water), with Gr=30 , $\lambda = -1.22$, pr = 1 and M=3.and,11a(Al2O3-water) Gr=20 , $\lambda = -1.22$, pr = 6.2 and M=5 ,increasing the solid nanoparticle volume fraction results in less elevation of velocity curves, the increased volume fraction of solid nanoparticles in the base fluid causes this behavior, which results in transient and steady-state profiles for the fluid velocity. Figures 10b and 11b show that an increase in solid nanoparticles has a minor effect on temperature (cu-water) and a simple increase in the effect of (Al2O3-water). Figs, 12(a,b)(cu-water) ,impacts of Pr =0.1,3,6.2 with $\varphi=0.2$, $\lambda = 1.22$, Gr = 20 and M=3 ,and figs13(a,b) (Al2O3-water),impacts of Pr =0.04,3,12 with $\varphi=0.2$, $\lambda=1.22$, and Gr = M=3,these two graphs (Figs. 12 and 13) show the effects of the Prandtl number Pr on the velocity and temperature profiles, since Pr is the ratio of momentum diffusivity to thermal diffusivity, larger values of Pr result in a significant decrease in both velocity and temperature profiles, according to Figs.



Figs. 2a,2b, :Velocity and temperature profiles influenced by magnetic parameter (M) for cu-water with $\varphi=0.2$, $pr = 6.2$, $\lambda=-1.22$, and $Gr= 3$.

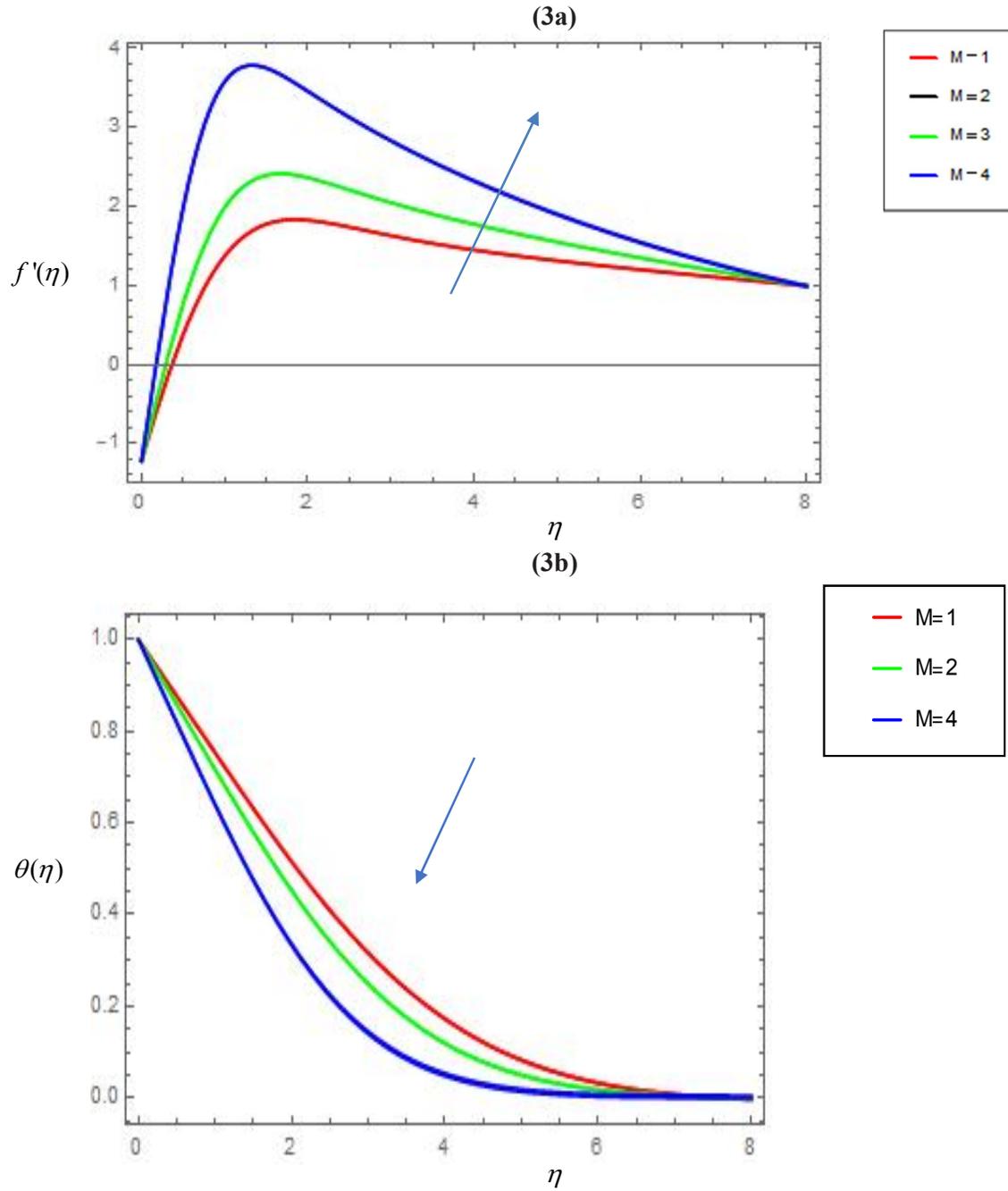
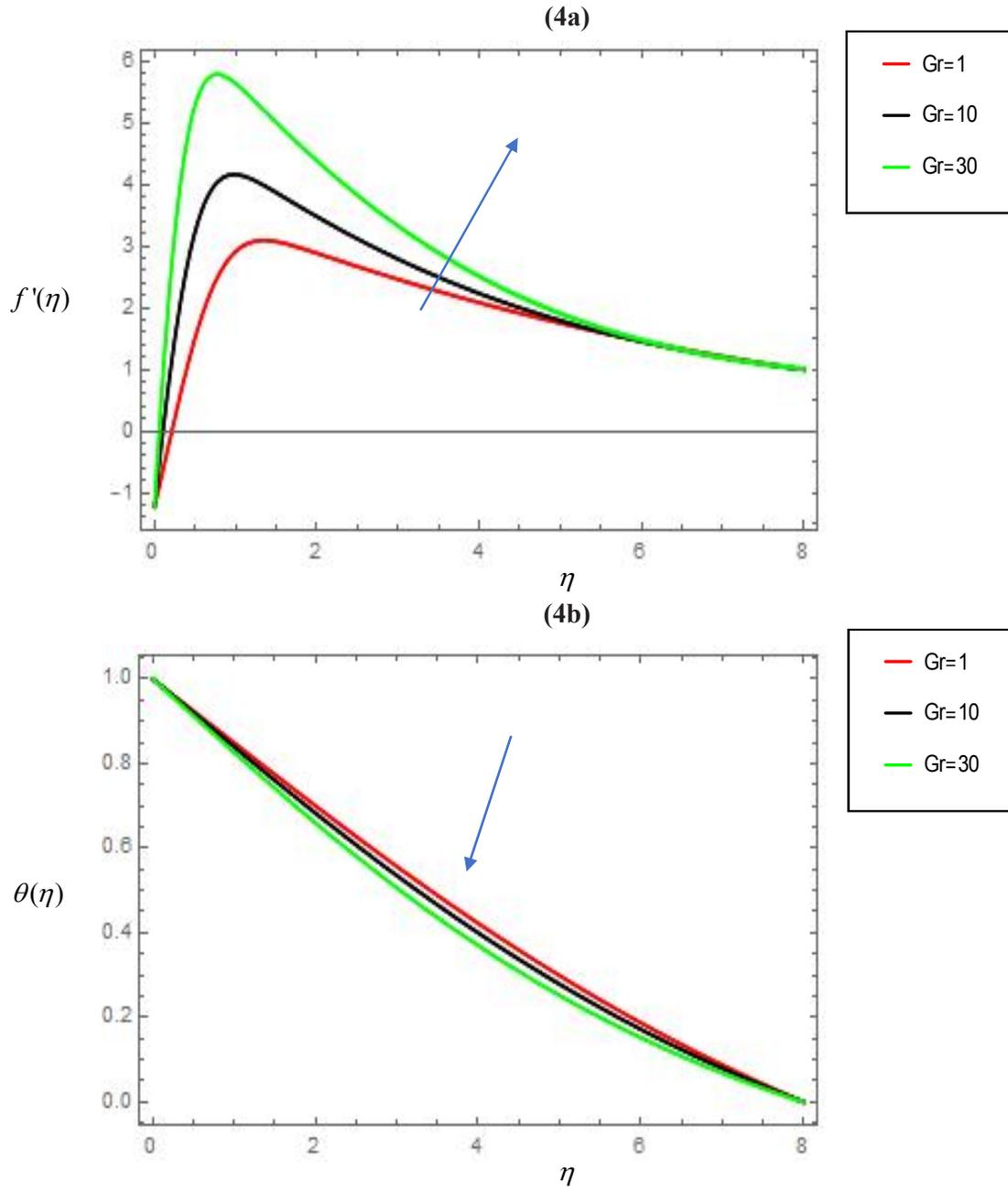
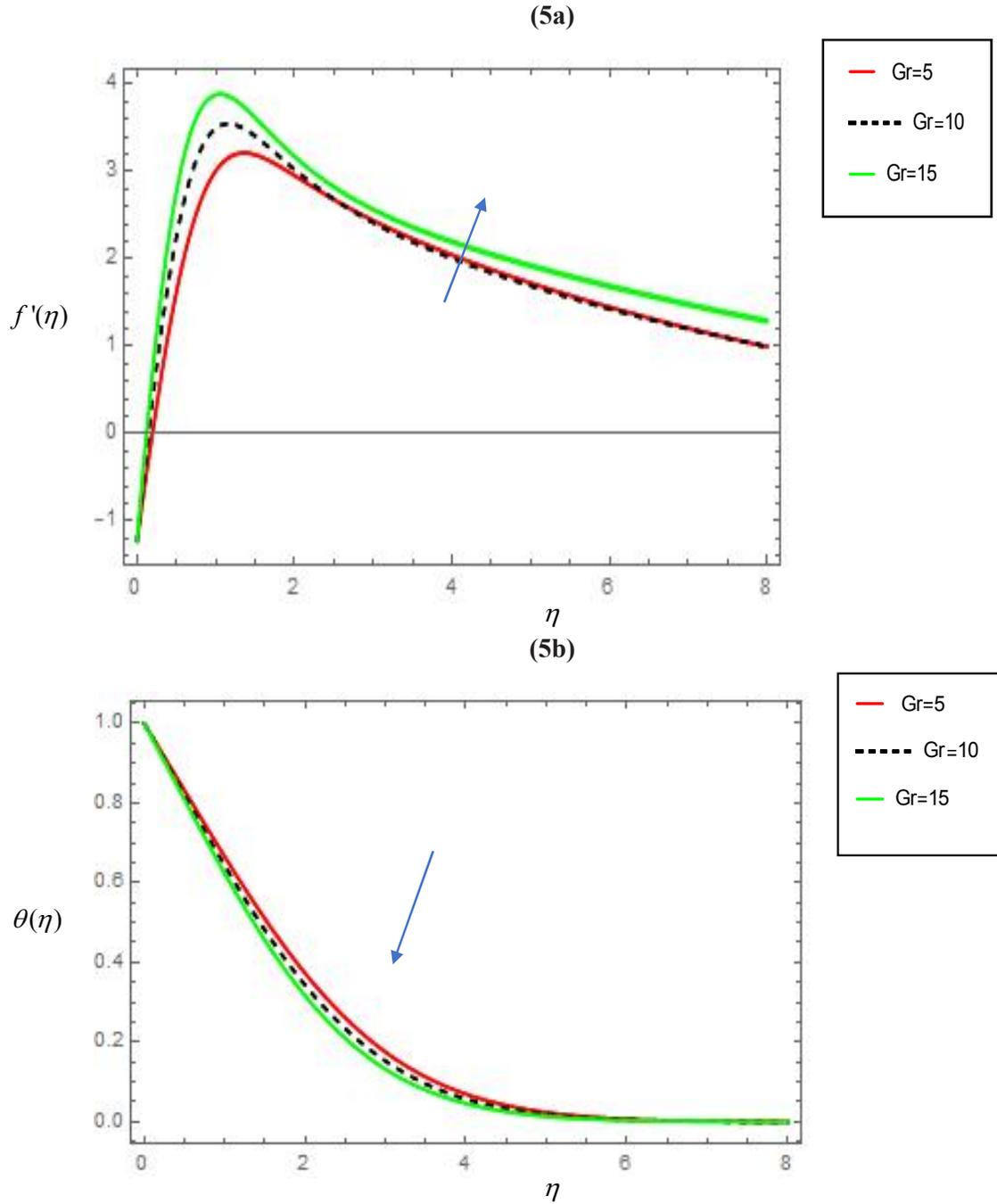


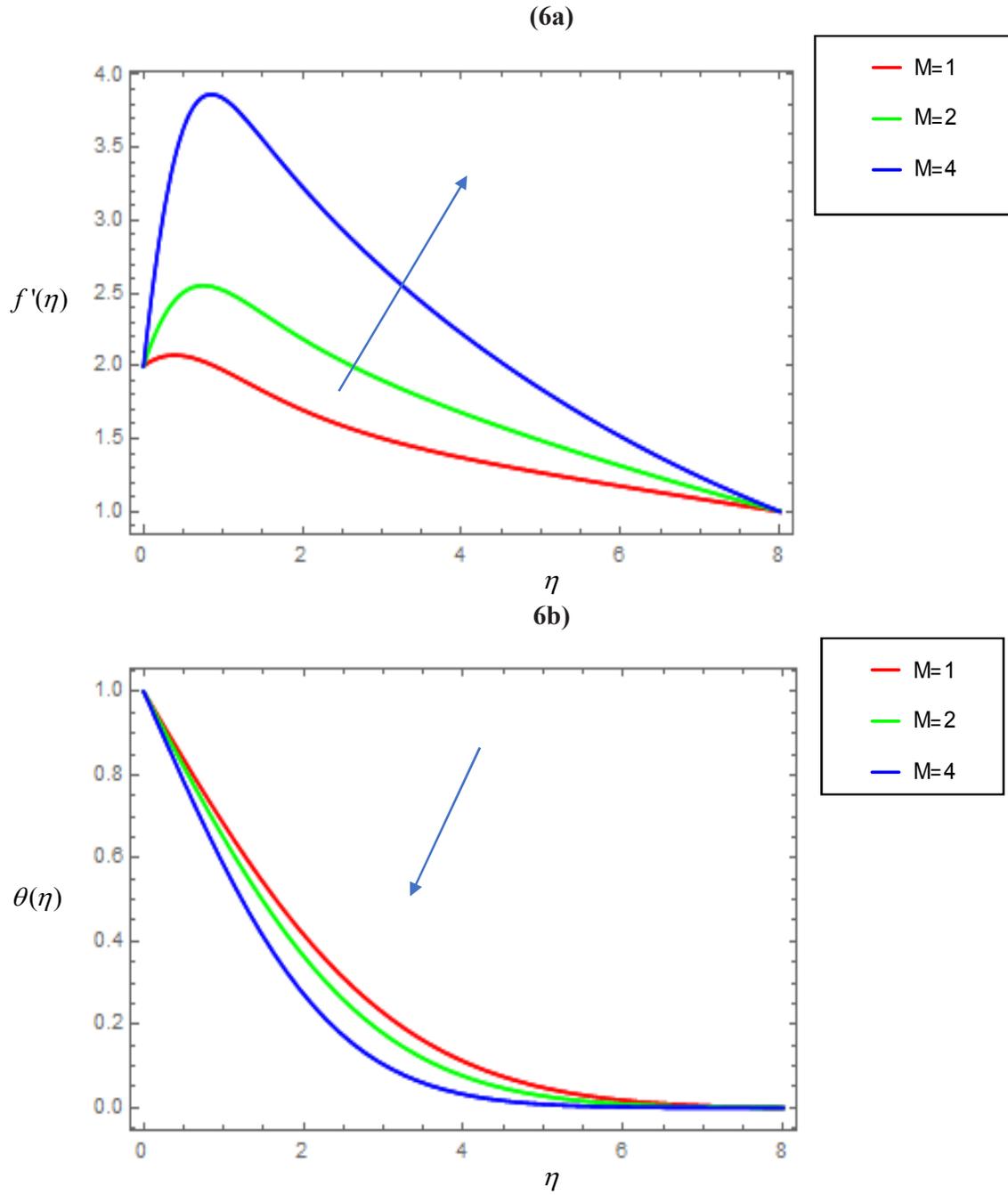
Fig. 3a and 3b: Shows the effect of the magnetic parameter (M) with various parameters for $(Al_2O_3-water)$ with, $\lambda = -1.22$, $\varphi = 0.2$, $pr = 6.2$ and $Gr = 3$.



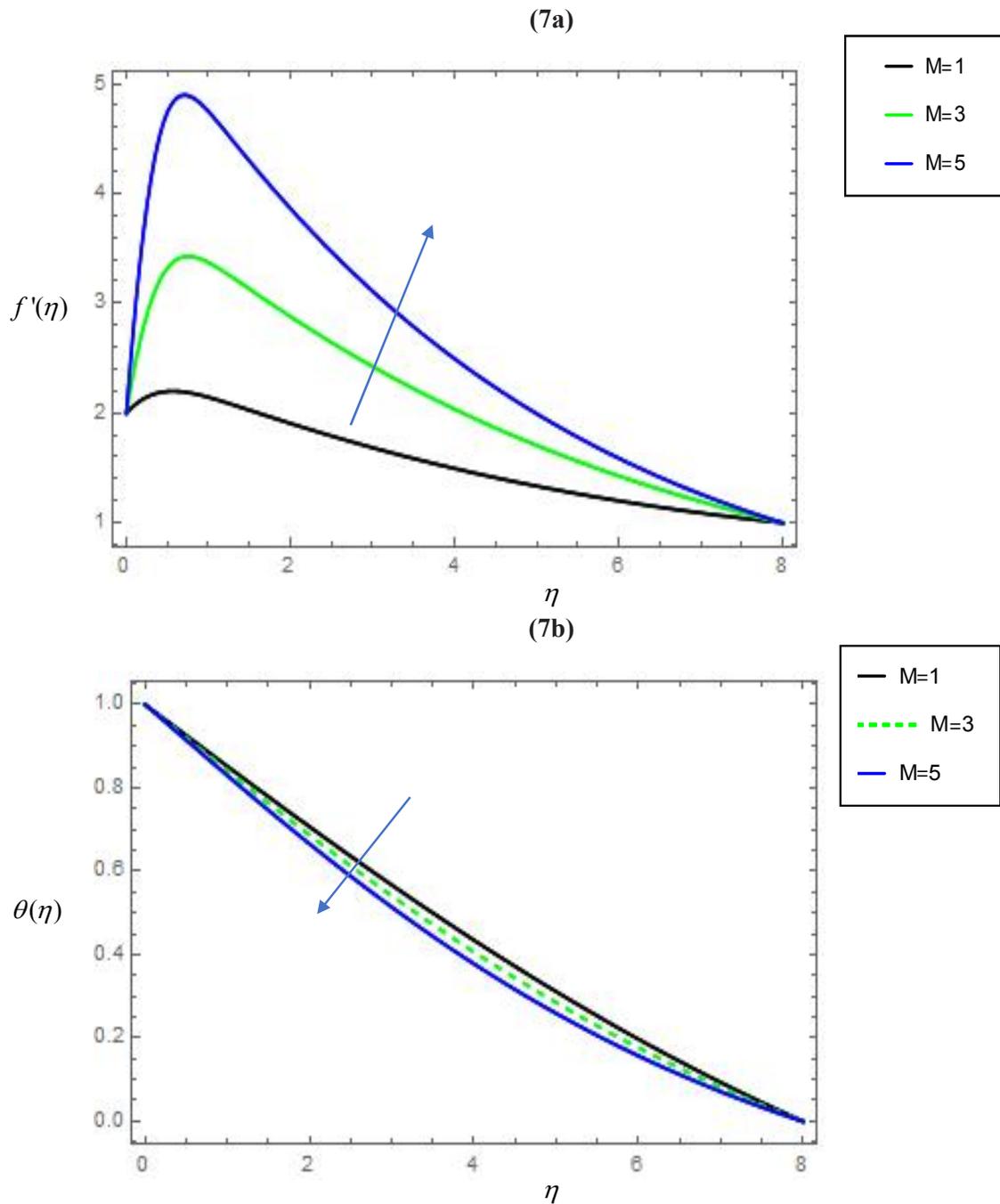
Figs 4a, 4b: Effect of grashof number (Gr) for cu-water with $\varphi=0.2$, $\lambda=-1.22$, $pr = 6.2$ and $M=3$.



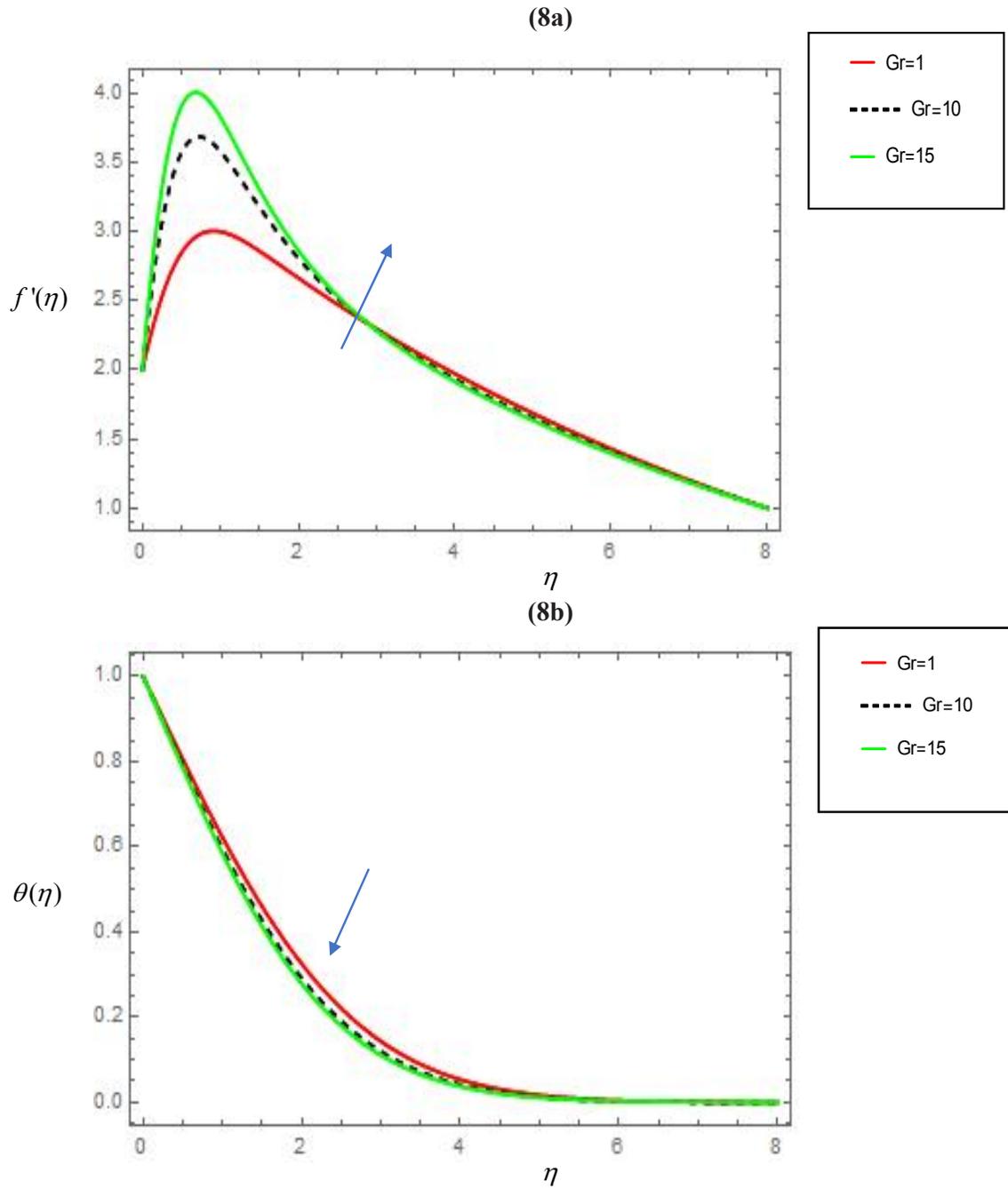
Figs 5a,5b: Shows the effect of Grshof number (Gr) for Al_2O_3 -water working fluid with $\phi=0.2$, $\lambda=-1.22$, $pr=6.2$ and $M=3$.



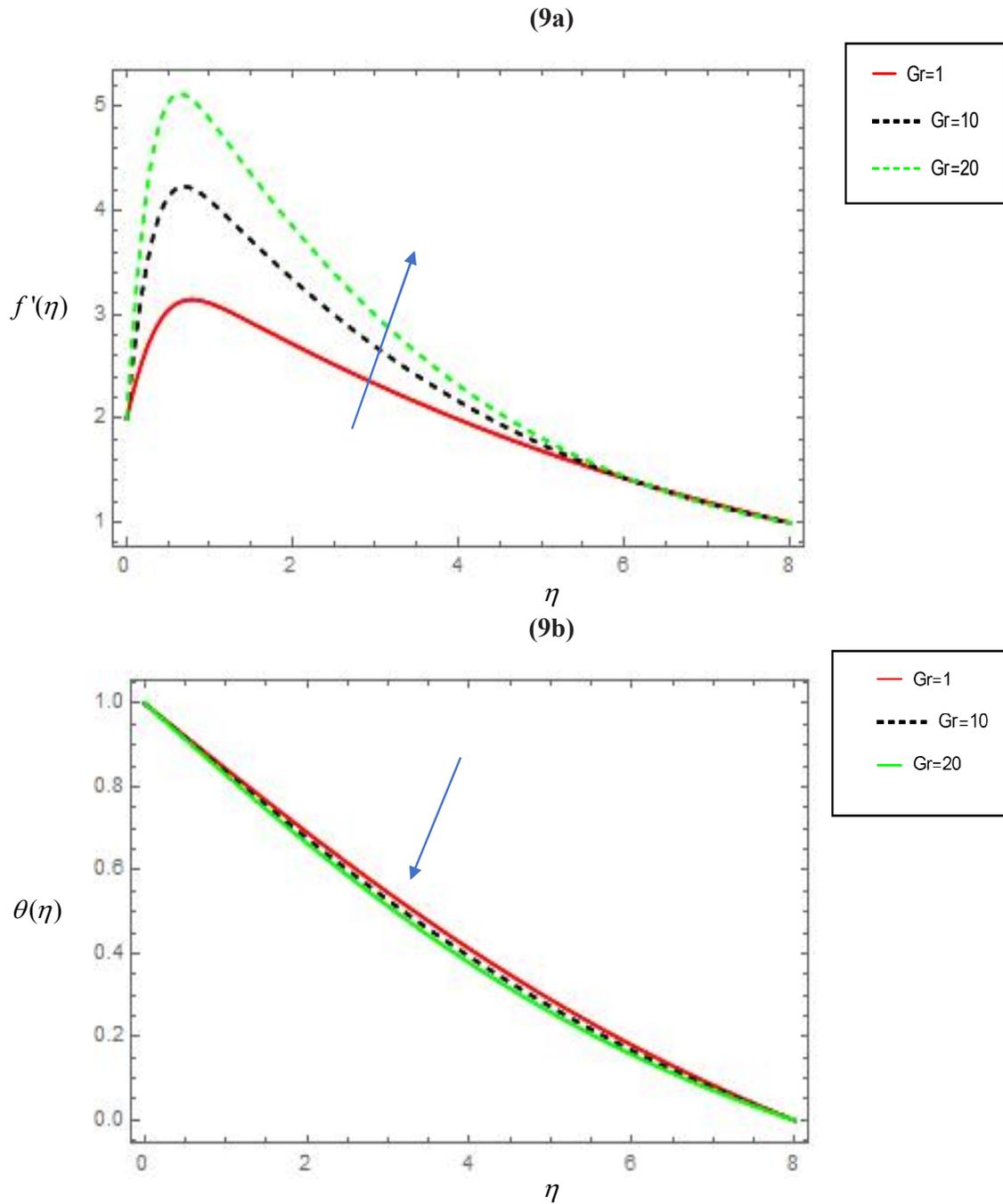
Figs 6a, 6b: Shows the effect of magnetic parameters (M), for Al_2O_3 -water working fluid with $\phi = 0.2$, $\lambda = 2$, $pr = 6.2$ and $Gr = 3$.



Figs 7a.7b: Impact of magnetic parameters in stretching for cu-water with $\lambda = 2$, $\phi = 0.2$, $pr = 6.2$ and $Gr = 3$.



Figs 8a, 8b: Impact of grashof number (Gr) for Al_2O_3 -water working fluid with $\varphi=0.2$, $\lambda=2$, $pr=6.2$ and $Gr=3$.



Figs 9a & 9b: Shows the impact of grashof number (Gr) in stretching for (cu-water) with $\lambda = 2$, $\phi = 0.2$, $pr = 6.2$ and $M = 3$.

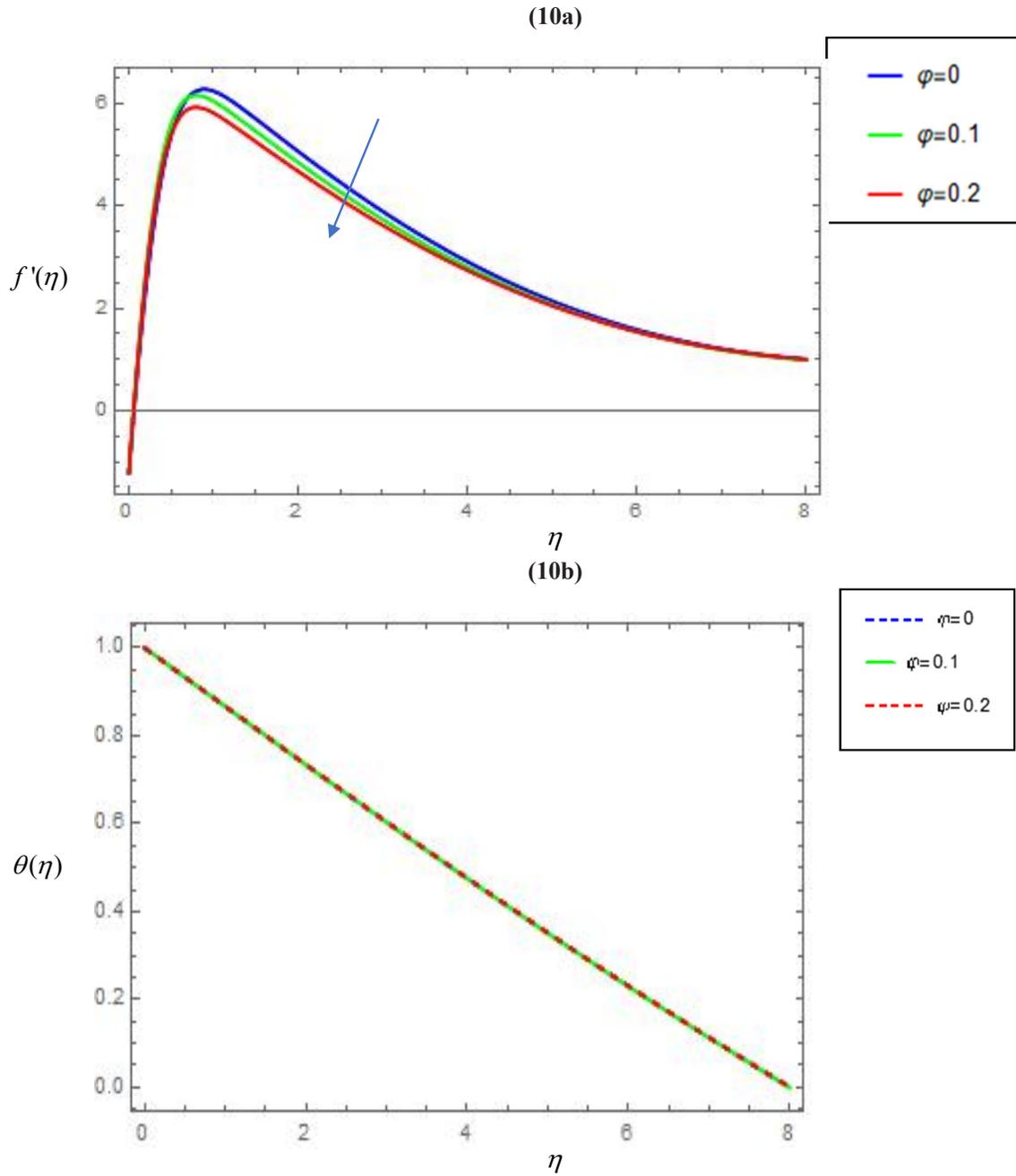
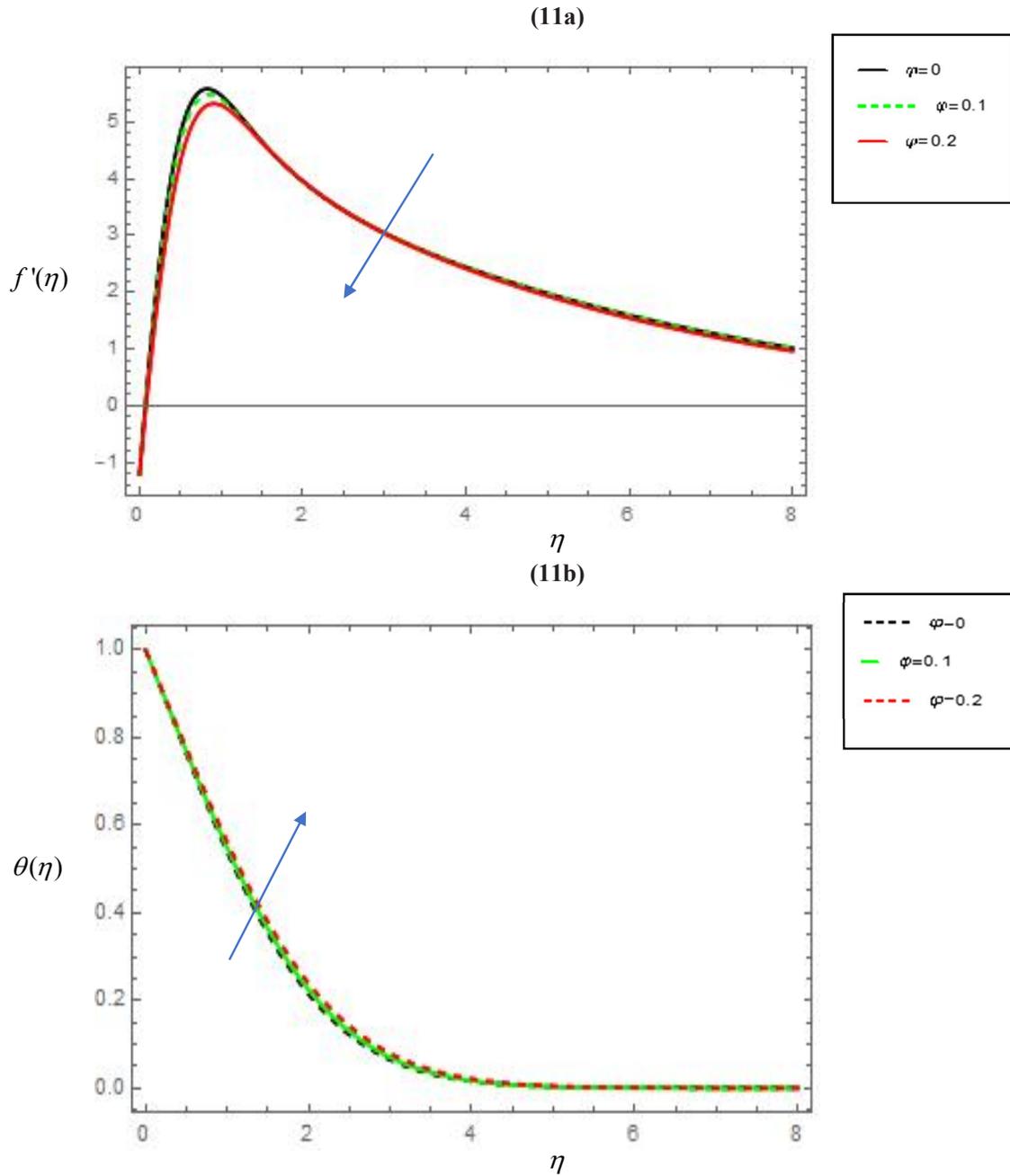
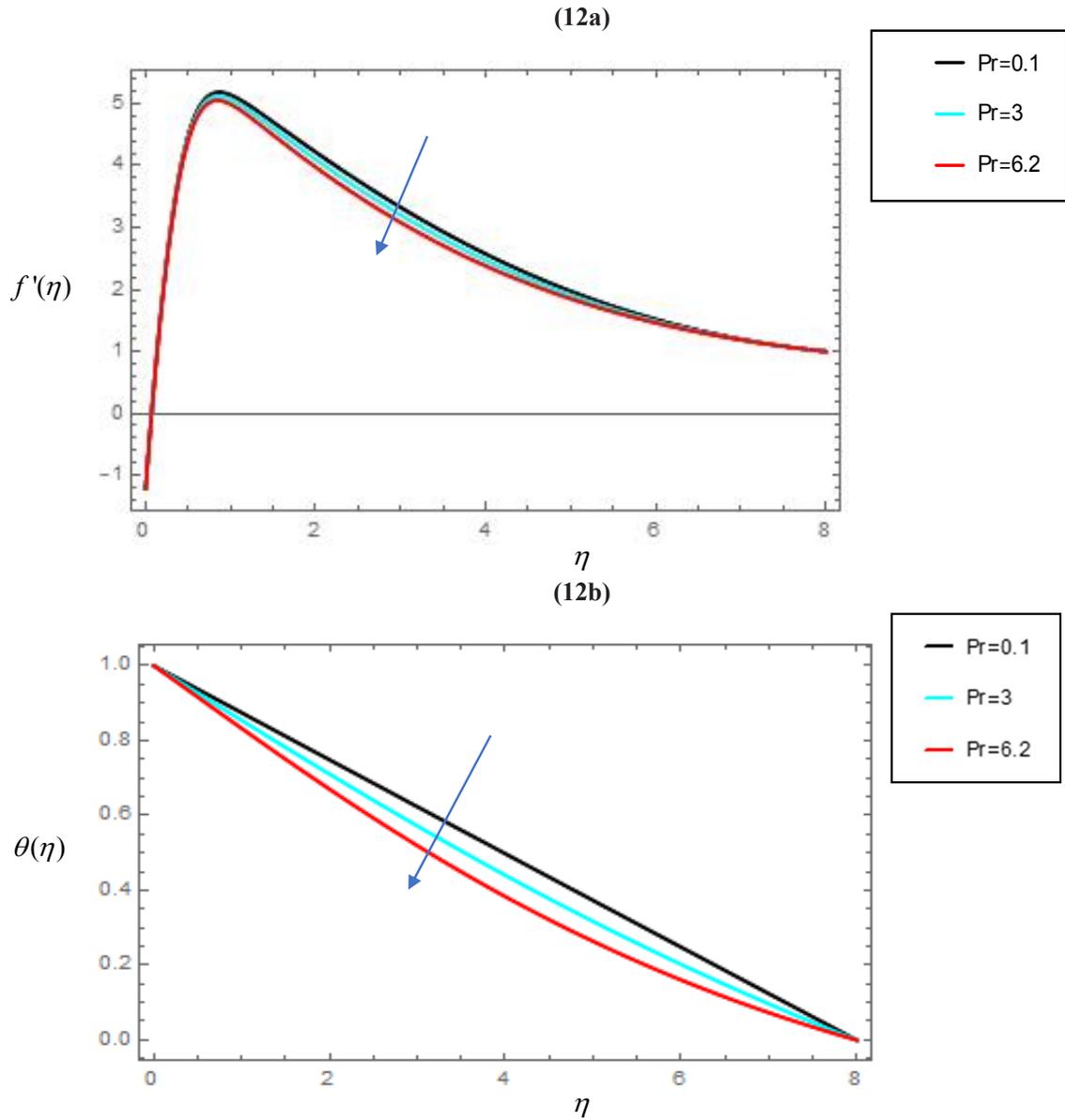


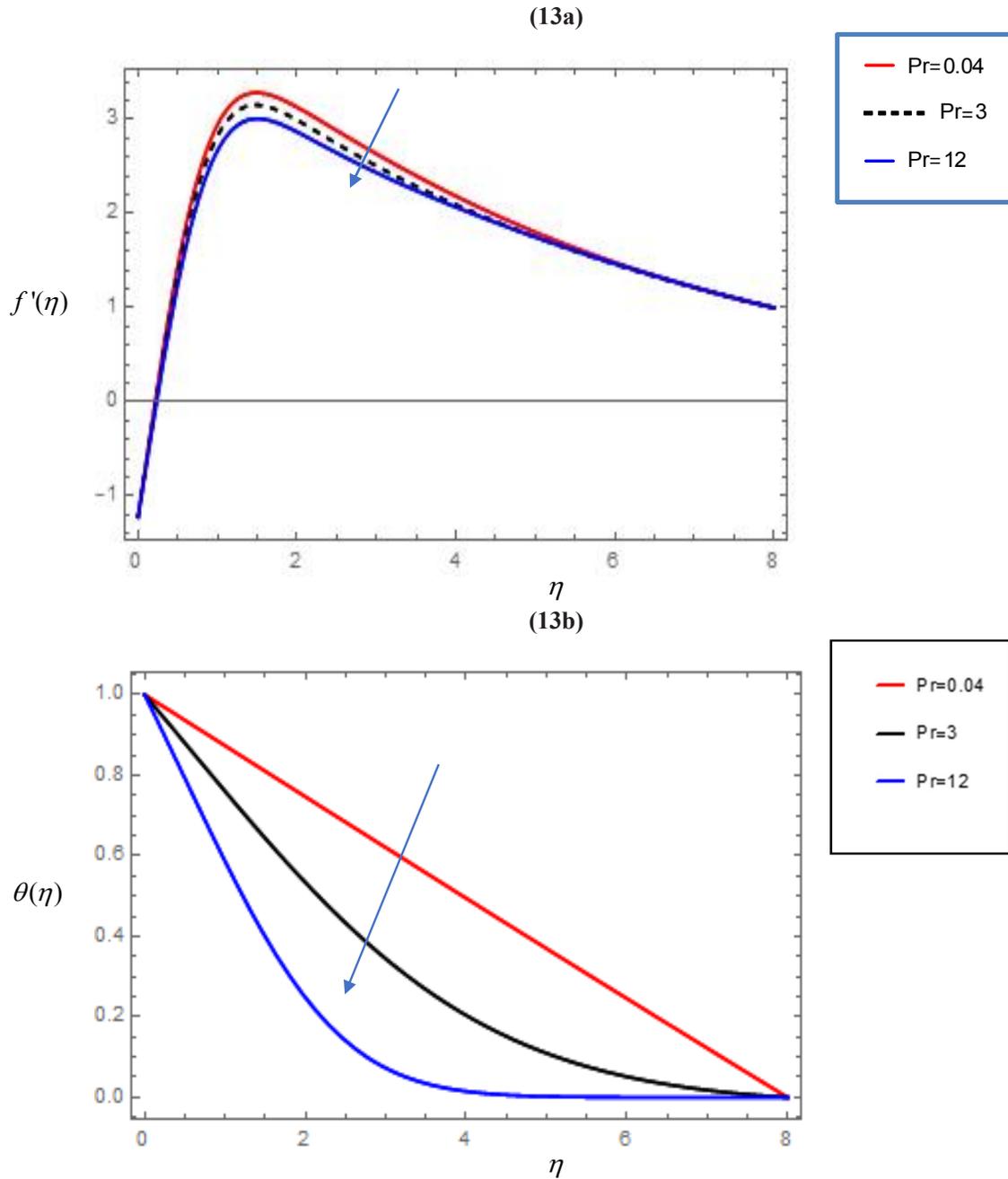
Fig. 10a and 10b: Shows the effect of solid volume on velocity and temperature with some values of φ ($0 \leq \varphi \leq 0.2$), for (Cu-water) with $\lambda = -1.22$, $M=3$, $Gr=30$ and $pr=1$.



Figs 11a&11b: Velocity and temperature profiles for some values of φ ($0 \leq \varphi \leq 0.2$) for (Al_2O_3 -water) with $\lambda = -1.22$, $\text{Gr}=20$, $M=5$ and $\text{Pr} = 6.2$.



Figs 12a&12b: Shows the influence of prandle number (pr) in cu-water fluid with $\phi=0.2$, $\lambda = -1.22$, Gr= 20, and M=3



Figs 13a&13b: Shows the impact of (pr) for (Al₂O₃-water) with $\varphi=0.2$, $\lambda = -1.22$, and $M=Gr=3$.

4. Conclusions:

The influence of a magnetic field and heat flux on a stretching/shrinking sheet in a nanofluid was studied in this article. For two types of nanoparticles, namely copper (Cu) and alumina (Al₂O₃), in a water-based fluid with Prandtl number $Pr = 6.2$, (PDEs) are converted to (ODEs) using a similarity transformation, solved numerically using a shooting method. The investigation of stretching/shrinking velocity and temperature profiles numerical results shows the effect of Grashof number (Gr), Magnetic parameter (M), Prandtl number (pr), and the solid volume fraction parameter (φ). It mentioned that the effects of the magnetic parameter (M) and Grashof number (Gr) for a shrinking/shrinking sheet increase velocity while decreasing temperature. For the shrinking sheet, we studied the impacts of nanoparticles on copper (Cu), which resulted in a decrease in velocity and a

minor effect on temperature, while in alumina (Al₂O₃-water), the velocity decreases appreciably with an increase in the nanoparticle volume fraction and simple increase effect in temperature, the highest prandle numbers reduce velocity and temperature profiles for copper and alumina.

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