Nonlinear Analysis of Quasi Static Consolidation for Fully Saturated Porous Media Subjected to Dynamic Loads

Ramadan Awadalla¹, W. Abbas², S. Bicher³, Mostafa A.M. Abdeem³ and E.S.M. El Shinnawy¹

¹Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt.
²Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt.
³Faculty of Engineering, Department of Engineering Mathematics and Physics, Cairo University, Giza, Egypt.

Received: 20 Sept. 2017 / Accepted: 28 Nov. 2017 / Publication date: 14 Dec. 2017

ABSTRACT

The behavior of geomaterials spatially in soils is governed by a set of partial differential equations caused by the interaction between solid skeleton and pore fluid. The present work, introduces the semi analytical solution of quasi static consolidation for fully saturated porous media subjected to dynamic loads. The governing equations with the corresponding boundary conditions solved analytically using the regular perturbation technique. The nonlinearity of material and geometry are considered. The mathematical formulation of the problem for the quasi-static consolidation is introduced and the parametric study is used to investigate the effect of nonlinearity of material and geometry on the pore pressure, and total stress.

Kay words: Porous media, Perturbation technique, Pore fluid, Total stress, Quasi static.

Introduction

Most of geotechnical engineering problems are focused on transient phenomena occurring in earthquakes, wave loading, and consolidation. All of these cases, the interaction between the deformation of solid skeleton of soil and pore fluid is of major importance. Also, the same behavior occurred in other area such as, biomechanics, the solid skeleton referred to porous bone structure and the pore fluid to the circulating bloods. The formulation of quasi static phenomena which describes the interaction between the solid and fluid phases was first established by Biot (1941), and he extended these formulations to include the dynamic response Biot (1956, 1962). At later date Truesdell (1957, 1960) introduced the mixture theory. For quasi static consolidation analysis, the assumption of drained and undrained behavior may be depending on the loading rapidity and permeability of soil. Zienkiewicz and Bettes (1980) presented linear quasi static formulation of two phase medium, based on the work of Boit. These formulation were applied to a simple problem of homogeneous soil layer under periodic loads and from the exact solution, the limits of various assumptions were good. After that, Zienkiewiczand Shiomi(1984) improved his formulation by inserting the nonlinear term of acceleration of pore fluid. The generalized two-phase behavior of soil was established by Zienkiewicz et al. (1990). The volumetric expansion of the solid-fluid phase was included. After that, Xikuandi Zienkiewicz(1992) presented the deformation of multiphase flow of porous media; he took the saturation of wetting fluid as a primary unknown of model. A fully coupled dynamic model for the analysis of water and air flow of two phases in the deformable porous media was introduced by Bernhard et al. (2001). Elgamal et al. (2002, 2003) introduced the development of the computational model for analysis of cyclic mobility; he took the shear deformation of cohesion-less soils into consideration. The two dimensional of dynamic consolidation for fully saturated soil was developmented by Hassanenand El-Hamalawi (2007). The large deformation of coupled dynamic and contact analysis of fully saturated porous media presented by Yonggang et al. (2013). Nedjar

Corresponding Author: Ramadan Awadalla, Faculty of Engineering (Mataria), Engineering Physics and Mathematics Department, Helwan University, Cairo, Egypt. E-mail: Eng_RAM2010@yahoo.com

847
Formulation of The problem:

The saturated porous media containing the solid grains and a viscous fluid, these two materials are called two phase porous media. Biot (1941, 1960) introduced the constitutive relations governing the behavior of saturated porous media with dynamic moving loads. However, Zienkiewicz and Shiomi (1984) enhanced these relations by adding the nonlinear part of material behaviors. Therefore, he introduced the total momentum equilibrium equation for the solid-fluid assembly as:

\[ \sigma_{ij} + \rho g_t = \rho \ddot{u}_i + \rho_f \left( \ddot{w}_i + \dot{w}_k \dot{w}_{k,i} \right) \]  

(1)

Where, \( \sigma_{ij} \) is the total stress, \( u \) is the soil grains displacement, \( w \) is the pore fluid displacement, \( \rho \) is the assembly density, \( \rho_f \) is the fluid phase density, \( g_t \) is the body force acceleration.

The equation of motion of pore pressure as:

\[ -p_t - R_i + \rho_f b_i = \rho_f \left( \ddot{u}_i + \left( \frac{\partial w_i}{\partial t} + \dot{w}_k \dot{w}_{k,i} \right) / n \right) \]  

(2)

Where, \( p \) is the pore pressure, \( n \) is the porosity, and \( R_i \) represents the viscous drag force which, assuming the validity of the Darcy seepage law given by Attia et al. (2014, 2015, 2016):

\[ k_{ij} R_i = w_i \]  

(3)

Where, \( k_{ij} \) define the generally anisotropic permeability coefficients. For isotropy these are conveniently changed by a single \( k \) value.

We ought to note that, the permeability may be defined as a function of strain and of external temperature as:

\[ k_{ij} = k_{ij} \left( (e_{ij} - e_{ij}^0), \mathbf{T} \right) \]

The constitutive relations for solid skeleton are given as:

\[ \dot{\sigma}_{ij} = \left( D_{ijkl} \dot{e}_{kl} - \dot{\sigma}_{ij} \right) + \dot{\omega}_{ik} \sigma_{kj} + \dot{\omega}_{jk} \sigma_{ki} - \alpha \dot{e}_{ij} \dot{p} \]  

(4)

Where \( \dot{e}_{ij} = (\dot{u}_{ij} + \dot{u}_{ji}) / 2 \), \( \dot{\omega}_{ij} = (\dot{u}_{ij} - \dot{u}_{ji}) / 2 \), \& \( \alpha \equiv (1 - K_T / \mathbf{T}) \leq 1 \)

Where, \( \dot{e}_{ij} \) is the rate of deformation tensor, \( \dot{\omega}_{ij} \) is the spin tensor, \( \dot{e}_{ij}^0 \) is the rate of thermal of other initial strain, \( D_{ijkl} \) is the tangent modulus matrix, and \( K_T \) is the bulk modulus of porous media.

The continuity equation of flow as:
\[ \dot{p}/Q + \alpha \dot{\varepsilon}_l + \dot{w}_l + \left( \dot{\rho}_f/\rho \right) w_l + \dot{s}_0 = 0 \]  
\hspace{1cm} (5)

Where, \( \dot{s}_0 \) represents the rate of volume change of fluid such as may be caused by thermal change, etc., the term \( Q \) represents the combined compressibility of fluid and solid phases, which can be defined by the relation containing the bulk modulus of each component as introduced by Biot and Willis (1957), Zienkiewicz (1982).

\[ 1/Q = n/K_S + (\alpha - n)/K_f \]  
\hspace{1cm} (6)

Where, \( K_S \) is the bulk modulus of solid phase, \( K_f \) is the bulk modulus of the fluid.

Figure 1 represents a formulation of fully saturated porous media subjected to dynamic loads, in this work, the set of partial differential equations for quasi-static of consolidation will be formulated in the vertical direction only, and the nonlinearity for material and geometry will be considered. In this case, the motion is slow phenomena; therefore, all acceleration terms for solid-fluid phases will be neglected. The perturbation technique will be used in order to solve this system of nonlinear partial differential equations.

![Image](image.png)

**Fig. 1:** Fully saturated porous media under dynamic loads

The quasi-static nonlinear partial differential equations become:

\[ \frac{\partial \sigma_{zz}(x,t)}{\partial x} = \rho_f \frac{\partial w(x,t)}{\partial t} \frac{\partial^2 w(x,t)}{\partial z \partial t} + \frac{\rho_f}{n} \frac{\partial p(x,t)}{\partial z} \]  
\hspace{1cm} (7)

\[ \frac{\partial \sigma_{zz}(x,t)}{\partial t} - \rho_f \frac{\partial w(x,t)}{\partial t} \frac{\partial^2 w(x,t)}{\partial z \partial t} + \frac{\rho_f}{n} \frac{\partial p(x,t)}{\partial z} = D(\alpha) \frac{\partial^4 u(x,t)}{\partial x^2 \partial t} \]  
\hspace{1cm} (8)

\[ \frac{\partial p(x,t)}{\partial t} = \alpha \frac{\partial^2 u(x,t)}{\partial z \partial t} + \frac{\partial^2 w(x,t)}{\partial z \partial t} \]  
\hspace{1cm} (9)

\[ \frac{\partial p(x,t)}{\partial z} = 0 \]  
\hspace{1cm} (10)

The boundary conditions of a fully saturated soil are:

At the surface \((z=0)\):

\[ p(0,t) = 0, \quad \sigma_{zz}(0,t) = q_e^{io\omega t}, \quad \frac{\partial u(0,t)}{\partial z} = \frac{q}{D_o} e^{io\omega t}, \text{ and } \frac{\partial w(0,t)}{\partial z} = -\frac{q}{D_o} e^{io\omega t} \]

At the base \((z=h)\):

\[ \frac{\partial p(h,t)}{\partial z} = 0, \quad \frac{\partial \sigma_{zz}(h,t)}{\partial z} = 0, \quad u(h,t) = 0, \quad \text{ and } w(h,t) = 0 \]
Where, \( q \) is the periodic load intensity, and \( D_o \) is the modulus of rigidity at the surface. 
We introduce the following non-dimensional form of parameters as:

\[
Z = \frac{z}{h}, \quad U = \frac{u}{(qh/D_o)}, \quad W = \frac{w}{(qh/D_o)}, \quad \sigma_{zz} = \frac{\sigma_{zz}}{q}, \quad p = \frac{p}{q}, \quad \beta = \frac{p_f}{\rho}, \quad \gamma = \frac{\beta}{n},
\]

\[
T = \frac{t}{h} V_c
\]

Where, \( V_c \) is the compression wave velocity, and given by \( V_c^2 = \frac{D_o + q}{\rho} \).

Assume the nonlinear rigidity of solid skeleton \( D(z) \) take the form:

\[
D(z) = D_o \left[ 1 + \left( \frac{q}{D_o} \right) f(z) \right]^2
\]

By using the non-dimensional quantities into the set of equations (7) through (10) get:

\[
(1 - K_1) \frac{\partial \sigma_{zz}(Z,T)}{\partial z} = \beta \left( \frac{\partial}{\partial z} \frac{\partial w(Z,T)}{\partial T} \right) + \gamma \left( \frac{\partial}{\partial z} \frac{\partial^2 w(Z,T)}{\partial T^2} \right)
\]

\[
- (1 - K_1) \frac{\partial \sigma_{zz}(Z,T)}{\partial T} = \frac{q}{D_o} f(z) \left( \frac{\partial^2 w(Z,T)}{\partial T^2} \right)
\]

\[
K_1 \left[ \frac{\partial^2 u(Z,T)}{\partial z \partial T} + 2 \frac{\partial^2 w(Z,T)}{\partial z \partial T} \right]
\]

Where, \( K_1 = \frac{q}{D_o + q} \), \( \pi_1 = \frac{kv_c^2}{\beta \omega h^2} \), \( \pi_2 = \frac{\omega^2 h^2}{V_c^2} \), and \( \tilde{\omega} = \pi_2^{1/2} \)

The regular perturbation method:

It is clear that in equations (11) through (14), the nonlinear part multiplied by the parameter \( q/D_o \), this parameter is very sensitive and called \( \zeta \), for the cases of engineering interest e.g. a tractor on a clay field, the value of \( \zeta \) ranges from 0.02 to 0.05. This value creates a weakly nonlinear problem. Therefore, we can use the regular perturbation method which is presented by Farlow (1982) to solve the formulated system of nonlinear partial differential equations:

\[
U(Z,T) = U_0(Z,T) + \zeta U_1(Z,T) + 0[\zeta^2 U_2(Z,T) + \zeta^3 U_3(Z,T) + \ldots]
\]

\[
W(Z,T) = W_0(Z,T) + \zeta W_1(Z,T) + 0[\zeta^2 W_2(Z,T) + \zeta^3 W_3(Z,T) + \ldots]
\]

\[
P(Z,T) = P_0(Z,T) + \zeta P_1(Z,T) + 0[\zeta^2 P_2(Z,T) + \zeta^3 P_3(Z,T) + \ldots]
\]

\[
\sigma_{zz}(Z,T) = \sigma_{zz0}(Z,T) + \zeta \sigma_{zz1}(Z,T) + 0[\zeta^2 \sigma_{zz2}(Z,T) + \zeta^3 \sigma_{zz3}(Z,T) + \ldots]
\]

From the above definition of the perturbation method, the zero perturbation equations become:

\[
(1 - K_1) \frac{\partial \sigma_{zz0}(Z,T)}{\partial z} = 0
\]

\[
- (1 - K_1) \frac{\partial \sigma_{zz0}(Z,T)}{\partial T} = \frac{1}{\pi_1 \pi_2} \frac{\partial \sigma_{zz0}(Z,T)}{\partial T}
\]

\[
\frac{\partial \sigma_{zz0}(Z,T)}{\partial T} = \frac{\partial^2 u_0(Z,T)}{\partial z \partial T} - \frac{\partial^2 w_0(Z,T)}{\partial z \partial T}
\]

\[
K_1 \left[ \frac{\partial^2 u_0(Z,T)}{\partial z \partial T} + \frac{\partial^2 w_0(Z,T)}{\partial z \partial T} \right]
\]

For a periodic loading, the solutions can be separated as:

\[
U_0(Z,T) = U_0(Z)e^{i\omega T}, \quad W_0(Z,T) = W_0(Z)e^{i\omega T}
\]

\[
\sigma_{zz0}(Z,T) = \sigma_{zz0}(Z)e^{i\omega T}, \quad P_0(Z,T) = P_0(Z)e^{i\omega T}
\]

Therefore, equations (16) through (19) may be written as:
\( (1 - K_1) \frac{d^2 \sigma_{zz}(Z)}{dZ^2} e^{i \omega \tau} = 0 \) \hfill (21)

\(- (1 - K_1) \frac{d P_0(Z)}{dZ} e^{i \omega \tau} = i \omega W_0(Z) e^{i \omega \tau} \) \hfill (22)

\( i \omega \sigma_{zz}(Z) e^{i \omega \tau} = i \omega \frac{d U_0(Z)}{dZ} e^{-i \omega \tau} - i \omega P_0(Z) e^{i \omega \tau} \) \hfill (23)

\(- i \omega P_0(Z) e^{i \omega \tau} = \frac{K_1}{1 - K_1} i \omega \left[ \frac{d U_0(Z)}{dZ} + \frac{d W_0(Z)}{dZ} \right] e^{i \omega \tau} \) \hfill (24)

The Non-dimensional boundary conditions for zero perturbation equations become:

At the surface \((Z = 0)\):

\[ P_0(0) = 0 , \quad \sigma_{zz}(0) = 1 , \quad dU_0(0)/dZ = 1 , \quad dW_0(0)/dZ = -1 \]

At the base \((Z = 1)\):

\[ dP_0(1)/dZ = 0 , \quad d\sigma_{zz}(1)/dZ = 0 , \quad U_0(1) = 0 , \quad W_0(1) = 0 \] \hfill (25)

Solving the set of system differential equations (21) through (24), the corresponding coupled second order differential equations may be obtained as:

The two equations of zero perturbation will be in these forms:

\[ \left[ \frac{d^2}{dZ^2} \right] U_0(Z) + \left[ K_1 \left( \frac{d^2}{dZ^2} \right) \right] W_0(Z) = 0 \] \hfill (26)

\[ [K_1(\alpha^2/dZ^2)]U_0(Z) + [K_1(\alpha^2/dZ^2) - (i/\pi_1)]W_0(Z) = 0 \] \hfill (27)

The full solutions of zero perturbation equations become:

\[ U_0(Z) = \frac{(1 - K_1)(Z - 1) + K_1 \sinh \left( a_0(Z - 1) \right)}{(a_0 \cosh(a_0))} \] \hfill (28)

\[ W_0(Z) = -\frac{\sinh \left( a_0(Z - 1) \right)}{(a_0 \cosh(a_0))} \] \hfill (29)

\[ P_0(Z) = -K_1 \left[ 1 - \cosh(a_0(Z - 1)) / \cosh(a_0) \right] \] \hfill (30)

\[ \sigma_{zz}(Z) = 1 \]

Where, \( a_0 = \sqrt{(i/\pi_1) / [K_1(1 - K_1)]} \)

From the definition of the perturbation method in equation (15), the first perturbation equations become:

\[ (1 - K_1) \frac{\partial \sigma_{zz}(Z, \tau)}{\partial Z} = \beta \left( \frac{\partial W_0(Z, \tau)}{\partial \tau} \right) \frac{\partial^2 W_0(Z, \tau)}{\partial Z \partial \tau} \] \hfill (31)

\[ -(1 - K_1) \frac{\partial P_0(Z, \tau)}{\partial Z} = \gamma \left( \frac{\partial W_0(Z, \tau)}{\partial \tau} \right) \frac{\partial^2 W_0(Z, \tau)}{\partial Z \partial \tau} + \frac{1}{\pi_1 \sqrt{\pi_1}} \frac{\partial W_0(Z, \tau)}{\partial \tau} \] \hfill (32)

\[ \frac{\partial^2 \sigma_{zz}(Z, \tau)}{\partial Z \partial \tau} = \frac{\partial^2 U_0(Z, \tau)}{\partial Z \partial \tau} + f(Z) \frac{\partial^2 W_0(Z, \tau)}{\partial Z \partial \tau} - \frac{\partial P_0(Z, \tau)}{\partial \tau} \] \hfill (33)

\[ \frac{\partial \sigma_{zz}(Z, \tau)}{\partial \tau} = \frac{K_1}{1 - K_1} \left( \frac{\partial^2 U_0(Z, \tau)}{\partial Z \partial \tau} + \frac{\partial^2 W_0(Z, \tau)}{\partial Z \partial \tau} \right) \] \hfill (34)

The two equations of first perturbation will be taking the form:

\[ \left[ \frac{d^2}{dZ^2} \right] U_1(Z) + K_1 \left( \frac{d^2}{dZ^2} \right) W_1(Z) = -(1 - K_1) \frac{d}{dZ} \left( f(Z) \frac{d U_0(Z)}{dZ} \right) \] \hfill (35)

\[ -\beta \pi_1 W_0(Z) dW_0(Z)/dZ \]

\[ [K_1(\alpha^2/dZ^2)]U_1(Z) + [K_1(\alpha^2/dZ^2) - (2i/\pi_1)]W_1(Z) = -\gamma \pi_2 W_0(Z) dW_0(Z)/dZ \] \hfill (36)

The full solutions of the first perturbation equations become:
Let, \( f(Z) = \sin Z \), and \( \alpha_1 = \sqrt{\frac{(\frac{1}{2})}{K_1(1-K_1)}} \)

\[
U_1(Z) = A_{11} + A_{12}Z + A_{13}\cosh(\alpha_1Z) + A_{14}\sinh(\alpha_1Z) + B_{1UV} \cosh(iZ) + B_{2UV}\cosh((\alpha_0 + i)Z - \alpha_0) + B_{3UV}\cosh((\alpha_0 - i)Z - \alpha_0) + C_{1UV}\sinh 2\alpha_0(Z - 1) \tag{37}
\]

\[
W_1(Z) = A_{15} + A_{16}Z + A_{17}\cosh(\alpha_1Z) + A_{18}\sinh(\alpha_1Z) + B_{1WV} \cosh(iZ) + B_{2WV}\cosh((\alpha_0 + i)Z - \alpha_0) + B_{3WV}\cosh((\alpha_0 - i)Z - \alpha_0) + C_{1WV}\sinh 2\alpha_0(Z - 1) \tag{38}
\]

\[
P_1(Z) = \frac{-K_4}{1-K_1} \{ (A_{12} + A_{16}) + \alpha_1(\alpha_1(A_{13} + A_{17}) \sinh(\alpha_1Z) + \alpha_1(A_{14} + A_{18}) \cosh(\alpha_1Z)) + \alpha_0 (i)(B_{1UV} + B_{1WV}) \sinh(iZ) + ((\alpha_0 + i)(B_{2UV} + B_{2WV}) \sinh((\alpha_0 + i)Z - \alpha_0) + (\alpha_0 - i)(B_{3UV} + B_{3WV}) \sinh((\alpha_0 - i)Z - \alpha_0) + (2\alpha_0)(C_{1UV} + C_{1WV}) \cosh(2\alpha_0(Z - 1)) \} \tag{39}
\]

\[
\sigma_{zz}(Z) = \frac{1}{1-K_1} \{ (A_{12} + K_4A_{16}) + \alpha_1(\alpha_1(A_{13} + K_4A_{17}) \sinh(\alpha_1Z) + \alpha_1(A_{14} + K_4A_{18}) \cosh(\alpha_1Z)) + \alpha_0 (i)(B_{1UV} + K_4B_{1WV}) \sinh(iZ) + ((\alpha_0 + i)(B_{2UV} + K_4B_{2WV}) \sinh((\alpha_0 + i)Z - \alpha_0) + (\alpha_0 - i)(B_{3UV} + K_4B_{3WV}) \sinh((\alpha_0 - i)Z - \alpha_0) + (2\alpha_0)(C_{1UV} + K_4C_{1WV}) \cosh(2\alpha_0(Z - 1)) \} + \sin(Z) [1 - K_1 + K_1 \cosh(\alpha_0(Z - 1))/\cosh(\alpha_0)] \tag{40}
\]

Where,

\[
A_{11} = -A_{12} - A_{13} \cosh \alpha_1 - A_{14} \sinh \alpha_1 + N_1, \quad A_{12} = -(N_4/\alpha_1^2) + N_2
\]

\[
A_{13} = -(N_4 \sinh \alpha_1/\alpha_1 + N_3) / \alpha_1^2 \cosh \alpha_1, \quad A_{14} = N_4/\alpha_1^3
\]

\[
A_{15} = -A_{16} - A_{17} \cosh \alpha_1 - A_{18} \sinh \alpha_1 + N_5, \quad A_{16} = -(N_5/\alpha_1^2) + N_6
\]

\[
A_{17} = -(N_5 \sinh \alpha_1/\alpha_1 + N_3) / \alpha_1^2 \cosh \alpha_1, \quad A_{18} = N_5/\alpha_1^3
\]

\[
B_{1UV} = A_1/ [K_1(1-K_1)(\alpha_0+i)^4 - (2i/\pi_1)(\alpha_0+i)^2]
\]

\[
B_{2UV} = A_2/ [K_1(1-K_1)(\alpha_0+i)^4 - (2i/\pi_1)(\alpha_0+i)^2]
\]

\[
B_{3UV} = a_3/ [K_1(1-K_1)(\alpha_0-i)^4 - (2i/\pi_1)(\alpha_0-i)^2]
\]

\[
B_{1WV} = a_5/ [K_1(1-K_1)(\alpha_0+i)^4 - (2i/\pi_1)(\alpha_0+i)^2]
\]

\[
B_{2WV} = a_6/ [K_1(1-K_1)(\alpha_0+i)^4 - (2i/\pi_1)(\alpha_0+i)^2]
\]

\[
B_{3WV} = a_7/ [K_1(1-K_1)(\alpha_0-i)^4 - (2i/\pi_1)(\alpha_0-i)^2]
\]

\[
C_{1UV} = b_1/ [K_1(1-K_1)(2\alpha_0)^4 - (2i/\pi_1)(2\alpha_0)^2]
\]

\[
C_{1WV} = b_2/ [K_1(1-K_1)(2\alpha_0)^4 - (2i/\pi_1)(2\alpha_0)^2]
\]

\[
\begin{align*}
\alpha_1 & = (1 - K_1)^2[K_1 + 2i/\pi_1] \\
\alpha_2 & = (1 - i\alpha_0)(1 - K_1)[-K_1(\alpha_0 + i) + 2i/\pi_1] K_1/2 \cosh \alpha_0 \\
\alpha_3 & = (1 + i\alpha_0)(1 - K_1)[-K_1(\alpha_0 - i) + 2i/\pi_1] K_1/2 \cosh \alpha_0 \\
\alpha_4 & = K_1(1 - K_1)^2 \\
\alpha_5 & = -K_1(1 - i\alpha_0)(1 - K_1)(\alpha_0 + i)^2 K_1^2/2 \cosh \alpha_0 \\
\alpha_6 & = -K_1(1 - i\alpha_0)(1 - K_1)(\alpha_0 + i)^2 K_1^2/2 \cosh \alpha_0 \\
\alpha_7 & = -K_1(1 + i\alpha_0)(1 - K_1)(\alpha_0 - i)^2 K_1^2/2 \cosh \alpha_0 \\
b_1 & = [-K_1\pi_2(\beta - \gamma)(2\alpha_0)^3 + 2\beta\pi_2(1/\pi_1)]/2\alpha_0 \cosh \alpha_0 \\
b_2 & = [\pi_2(2K_1\beta - \gamma)(2\alpha_0)^2]/2\alpha_0 \cosh \alpha_0 \\
N_1 & = -\cosh(i)[B_{1UV} + B_{2UV} + B_{3UV}] \\
N_2 & = \sinh(\alpha_0)[B_{2UV}(\alpha_0 + i) + B_{3UV}(\alpha_0 - i)] - C_{1UV}(2\alpha_0) \cosh(2\alpha_0) \\
N_3 & = -\cosh(i)[B_{1UV}(\alpha_0 + i)^2 + B_{2UV}(\alpha_0 + i)^2 + B_{3UV}(\alpha_0 - i)^2 - \cos(1)(1 - K_1 + K_1/\cosh(\alpha_0)] \\
N_4 & = \sinh(\alpha_0)[B_{2UV}(\alpha_0 + i)^3 + B_{3UV}(\alpha_0 - i)^3] - C_{1UV}(2\alpha_0)^3 \cosh(2\alpha_0) \\
N_5 & = -\cosh(i)[B_{1WV} + B_{2WV} + B_{3WV}] \\
N_6 & = \sinh(\alpha_0)[B_{2WV}(\alpha_0 + i) + B_{3WV}(\alpha_0 - i)] - C_{1WV}(2\alpha_0) \cosh(2\alpha_0) \\
N_7 & = -\cosh(i)[B_{1WV}(\alpha_0 + i)^2 + B_{2WV}(\alpha_0 + i)^2 + B_{3WV}(\alpha_0 - i)^2 + \cos(1)(1 - K_1 + K_1/\cosh(\alpha_0)] \\
N_8 & = \sinh(\alpha_0)[B_{2WV}(\alpha_0 + i)^3 + B_{3WV}(\alpha_0 - i)^3] - C_{1WV}(2\alpha_0)^3 \cosh(2\alpha_0) \\
\end{align*}
\]
\[-\pi_2 \left[ \left( \gamma - \beta \right) / \left( 1 - K_f \right) \right] + \gamma / K_f \left[ 1 + \tanh^2 \left( \alpha_0 \right) \right] + 2K_f \alpha_0 \tanh \left( \alpha_0 \right) \]

Substituting equations (28) through (30) and (38) through (40), into set of equations (15) yield:
\[
U(Z, T) = U_0(Z) \times \left\{ 1 + \zeta, \frac{U_1(Z)/U_0(Z)}{e^{ioT}} \right\} \times e^{ioT}
\]
\[
W(Z, T) = W_0(Z) \times \left\{ 1 + \zeta, \frac{W_1(Z)/W_0(Z)}{e^{ioT}} \right\} \times e^{ioT}
\]
\[
P(Z, T) = P_0(Z) \times \left\{ 1 + \zeta, \frac{P_1(Z)/P_0(Z)}{e^{ioT}} \right\} \times e^{ioT}
\]
\[
\sigma_{zz}(Z, T) = \sigma_{zz0}(Z) \times \left\{ 1 + \zeta, \frac{\sigma_{zz1}(Z)/\sigma_{zz0}(Z)}{e^{ioT}} \right\} \times e^{ioT}
\]

Result and Discussion

In the following sections, the analysis of the one dimensional fully saturated soil under periodic loads (complete case) are presented and discussed, all results are in good agreement with Zienkiewicz et al. (1980, 1999). The distribution of pore pressure, total stress, solid displacement, and fluid displacement with depth are introduced for many values of \( \pi_1 \) and \( \pi_2 \). Also, a comparison between linear and nonlinear cases is presented (including nonlinear rigidity). The values of parameters used in the results are:

\[
E = 3 \times 10^6 \text{ N/m}^2, \quad \nu = 0.2, \quad n = 0.3, \quad K_s = 10 \times 10^9 \text{ N/m}^2, \quad K_f = 4 \times 10^6 \text{ N/m}^2
\]
\[
Q = 120 \times 10^6 \text{ N/m}^2, \quad h = 10m, \quad \beta = 0.33, \quad \gamma = 1.11, \quad K_1 = 0.973, \quad \text{and} \quad T = 500
\]

Figures 2 through 5 illustrate the behavior of the fluid and solid skeleton displacements with the soil depth. From these figures, it is noticed that, the effect of the nonlinearity is very small due to the absence of the dynamic terms for slow phenomena. The nonlinearity slightly appears at higher values of \( \pi_1 \). Generally, the displacement is maximum near the surface and decreases away from the surface up to the base and vanished.

Figures 6 and 7 introduce the behavior of pore pressure with the soil depth for different values \( \pi_1 \) and \( \pi_2 \). It is clear that, the nonlinearity is affected by higher values \( \pi_1 \). The figures show the pressures are zero at the surface and increase gradually with the depth. It can be noticed that at low values of \( \pi_1 \) the pore pressure is almost constant except at surface it belongs to zero.

Figures 8 and 9 introduce the distribution of total stress against the depth. From these figures, it is noticed that, the normalized stress is constant and equal to one for linear analysis. Also, to low values of \( \pi_1 \) the nonlinear term has no effect on the analysis and results of linear and nonlinear are close to each other, while, with increasing the values of \( \pi_2 \) the effect of nonlinearity is noticeable specially at higher values of \( \pi_1 \).

![Fig. 2: Fluid displacement with depth, \( \pi_2 = 0.01 \)](image1.png)

![Fig. 3: Fluid displacement with depth, \( \pi_2 = 0.1 \)](image2.png)
In this study, a nonlinear analysis of fully saturated porous media is presented by formulating the decoupled nonlinear differential equations of quasi-static consolidation. The governing equations have been non-dimensional and solved analytically using the regular perturbation technique. The effect of nonlinearity of material and geometry on the solid and fluid displacements, the pore pressure, and total stress was considered. The parametric study of the analysis depends on two dimensionless parameters \( \pi_1 \) (function of permeability) and \( \pi_2 \) (function of frequency of applied load), and it was concluded that:

**Conclusion**

...
- At high values of frequencies of applied load, the geometric and material nonlinearity has a great effect on solid and fluid displacements, pore pressure, and total stress especially at higher values of permeability.
- The effect of nonlinearity is very weak due to the absence of the accelerations of solid and fluid for slow phenomena.
- At low level of permeability, almost the geometric nonlinearity has no effect on solid and fluid displacements, pore pressure, and total stress.

References


